

Of Fractions

A Fraction is that by which some part or parts of an unit are expressed.

Thus if an unit be supposed to be divided into 4 ~~different~~ ^{parts} equal parts and 3 of these parts are to be expressed it must be done by the fraction of three fourths or three quarters to be written thus in figures

$$\frac{3}{4}$$

The number which shows into how many parts the unit is

supposed to be divided is called the denominator of the fraction. Thus in the foregoing example 4 is the denominator of the fraction

$\frac{3}{4}$. The number which shows how many of these parts are considered in the fraction is called the numerator for thus 3 is the numerator of the fraction $\frac{3}{4}$.

So likewise in the fractions $\frac{5}{6}$, $\frac{3}{8}$, $\frac{9}{10}$, $\frac{50}{99}$ the denominators are 6, 8, 10, 99, & the Numerators, 5, 3, 9, 50.

Fractions are of two sorts, proper, and improper.

A proper fraction is that whose numerator is less than the denominator as $\frac{1}{2}$.

An improper fraction is that whose numerator is equal to or greater than the denominator as $\frac{2}{2}$, $\frac{3}{2}$ &c.

A mixed number is a whole number or integer & a fraction, such as $8\frac{1}{2}$.

To reduce a whole or mixed number to an ^{ro}improper fraction.

1st If a whole number be proposed multiply it by any proposed denominator and the product

will be the numerator of the
improper fraction requir'd
Ex. Let it be requir'd to reduce
8 to an improper fraction whose
denominator is 3.

Then $3 \times 8 = 24$ conseq. $8 = \frac{24}{3}$

Secondly if a mix'd number be
propos'd multiply its integer
by the denominator of the
fraction & to the product
add the numerator of the fraction
the sum will be the numerator
of the improper fraction
requir'd.

Ex. $8\frac{1}{2}$. $3 \times 8 = 24$ & $24 + 1 = 25$.

Ex. ~~$8\frac{1}{3}$~~

Therefore $\frac{25}{3} = 8\frac{1}{3}$

To reduce an improper fraction
to a whole or mixed number.

Divide the numerator by the
denominator the Quotient will
be the number required.

Ex. 1. $\frac{50}{7}$ $7 \overline{) 50} (7 \frac{1}{7}$ $7 \overline{) 50} (7 \frac{1}{7}$

$$\begin{array}{r} 7 \overline{) 50} \\ \underline{49} \\ 1 \end{array}$$

Ergo $\frac{50}{7} = 7 \frac{1}{7}$

Ex. 2. $\frac{207}{3}$ $3 \overline{) 207} (69$

$$\begin{array}{r} 3 \overline{) 207} \\ \underline{18} \\ 27 \\ \underline{27} \\ 0 \end{array}$$

Ergo $\frac{207}{3} = 69$

To reduce a fraction to
the lowest terms.

find the greatest common
divisor of the numerator &
denominator each of which being
divided by this divisor the
Quotient ~~is~~ will be the
numerator & denominator
of the fraction requir'd.

Ex. 36 the greatest common
divisor is 12 ergo $\frac{36}{12} = 3$

$\frac{120}{12} = 10$ ergo $\frac{3}{10} = \frac{36}{120}$
is the fraction requir'd.

(1794)

P. To find the greatest common divisor of two numbers proposed.

Divide the greatest by the least & if there be a remainder divide the least number by that remainder & if there be a second remainder divide the first remainder by it & so on till at last you come to some divisor which will divide the remainder immediately preceding exactly then will this last divisor be the greatest common divisor of the two given numbers.

Ex. Let 1344 & 582 be the num-
bers propos'd. then to find their
^{greatest} common divisor, divide 1344
by 582, there remains 180, then
divide 582 by 180, remains
42; divide 180 by 42, remains
12; divide 42 by 12, remains
6; lastly, divide 12 by 6, & there
remains nothing. Therefore
6, is the greatest com^{mon} divi^{sor}
of 1344 & 582.

A prime number is that which
has no other divisor than unity
as, 2, 3, 5, 7, &c.

If two numbers have no other
common divisor than unity, they

(1775)

• are said to be prime to each other; such as, 5 & 7; 9 & 14.

P. & To reduce different fractions to the same denomination.

1st. If two fractions be given multiply both the terms of the one by the denominator of the other.

Ex. To reduce ~~to the same~~ $\frac{2}{3}$ & $\frac{4}{5}$ to the same denomination.

$$\begin{array}{r} \frac{2}{3} \\ \hline 10 \\ \hline 15 \end{array} \qquad \begin{array}{r} \frac{4}{5} \\ \hline 12 \\ \hline 15 \end{array}$$

2^d. If three or more fractions be given multiply the terms of each by the product of the denominators of the rest.

Ex. $\frac{2}{3}, \frac{1}{6}, \frac{3}{4}$

$\frac{48}{72}$	$\frac{12}{72}$	$\frac{54}{72}$
-----------------	-----------------	-----------------

Ex. 3^d

$\frac{1}{3}$	$\frac{1}{5}$	$\frac{3}{7}$	$\frac{2}{9}$
315	189	405	210
945	567	1215	630

1776

Add Fractions

Reduce them to the same denomination & add the numerators.

Ex. 1st. $\frac{2}{5} + \frac{4}{5} = \frac{10}{15} + \frac{12}{15} = \frac{22}{15}$
 $= 1\frac{7}{15}$

Ex. 2^d. $\frac{2}{3} + \frac{1}{6} + \frac{3}{4} = \frac{48}{72} + \frac{12}{72} + \frac{54}{72}$
 $= \frac{114}{72} = \frac{142}{72} = 1\frac{7}{12}$

To subtract one Fraction from another

Reduce them to the same denomination & subtract one numerator from the other.

$$\text{Ex. 1}^{\text{st}} \quad \frac{2}{3} - \frac{3}{7} = \frac{14}{21} - \frac{9}{21} = \frac{5}{21}$$

$$\text{Ex. 2}^{\text{nd}} \quad \frac{13}{21} - \frac{11}{42} = \frac{26}{42} - \frac{11}{42} = \frac{15}{42} = \frac{5}{14}$$

$$\text{Ex. 3}^{\text{rd}} \quad \frac{17}{15} - \frac{4}{5} = \frac{17}{15} - \frac{12}{15} = \frac{22}{15} \\ + \frac{22}{15} - \frac{12}{15} = \frac{10}{15} = \frac{2}{3}$$

To Multiply one Fraction by another.

Multiply the numerator of the one by the numerator of the other which will give the numerator of the product & then multiply the denominator of one Fraction by the denominator of the other which will give the denominator of the

product.

Ex. 1st. Multiply $\frac{1}{2}$ by $\frac{2}{3} = \frac{2}{15}$ the product is $= \frac{1}{4}$.

Ex. 2nd. $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$.

Ex. 3rd. $6 \times \frac{2}{3} = \frac{12}{3} = 4$.

Ex. 4th. $\frac{2}{3} \times \frac{7}{8} = \frac{14}{24} = \frac{7}{12}$

Ex. 5th. $\frac{2}{3} \times \frac{5}{6} \times \frac{9}{10} = \frac{90}{180} = \frac{1}{2}$.

Ex. 6th. $\frac{2}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$

Ex. 7th. $6\frac{3}{4} + 7 = 47\frac{1}{4}$.

Ex. 8th. $3\frac{3}{4} + 2\frac{2}{3} = \frac{15}{4} + \frac{8}{3} = \frac{20}{12} + \frac{32}{12} = \frac{52}{12} = 4\frac{1}{3}$

Ex. 9th. $96\frac{1}{2} + 24\frac{1}{3} = 193\frac{1}{2} + 73\frac{1}{3} = \frac{14089}{6} = 2348\frac{1}{6}$

Ex. 10th. $\frac{4}{2} \times \frac{12}{3} = \frac{48}{6} \cdot \frac{4}{2} = 2 \cdot \frac{12}{3} = 4$
 $= \frac{4}{2} \times \frac{12}{3} = 2 \times 4 = 8 = \frac{48}{6}$

Division of Fractions

Rule.

Multiply the numerator of the dividend into the denominator of the divisor for a new numerator & multiply the other ~~two~~ numerator & denominator together for a new denominator.

Ex. 1st. Let $\frac{6}{35}$ be divided by $\frac{3}{7}$

$$\frac{3}{7} \overline{) \frac{6}{35}} \left(\frac{22}{105} = \frac{2}{5} \right. \text{ p. } \frac{3}{7} \times \frac{2}{5} = \frac{6}{35}$$

Ex. 2^d

$$\frac{5}{12} \overline{) \frac{20}{27}} \left(\frac{240}{135} = 1\frac{7}{9} \right.$$

Ex. 3^d

$$\frac{2}{11} : \frac{2}{5} \times \frac{5}{7} = \frac{2}{11} \left(\frac{70}{110} = \frac{7}{11} \right.$$

Ex. 4th. $20\frac{2}{5} : 2\frac{2}{5} =$

$$\frac{17}{5} \overline{) \frac{102}{5}} \left(\frac{102}{17} = 6 \right.$$

Ex. 5th. $40\frac{26}{63} \div 5\frac{3}{7} =$ Then $40\frac{26}{63} = \frac{2546}{63}$

GEO ADAM MRS 32 (1778)
 and $\frac{5}{7} = \frac{30}{7}$, Then $\frac{30}{7} \left(\frac{2546}{63} \left(\frac{17822}{2394} = 7 \frac{1064}{2394} = 7 \frac{4}{9} \right) \right)$

Ex. 6th $13 : \frac{5}{7}$. $\frac{5}{7} \Big) \frac{13}{1} \left(\frac{91}{5} \right)$
 $\frac{5}{7} \Big) \frac{91}{1} \left(\frac{181}{5} \right)$
 $\frac{3}{41}$
 $\frac{40}{1}$

Ex. 7th $\frac{5}{7} : 8 = \frac{5}{22}$.

Ex. 8th $\frac{10}{18} : \frac{1}{2} = \frac{98}{24} = 4$.

Ex. 9th $\frac{5}{7} : \frac{1}{2} = 2 \frac{1}{2} = \frac{10}{4}$

$\frac{5}{4} = 1 \frac{1}{4}$. $1 \frac{1}{4} : \frac{1}{2} = 1 \frac{1}{4} \times 2 = 2 \frac{1}{2}$.

Ex. 10th $\frac{7}{9} : \frac{3}{4} = \frac{28}{27} = 1 \frac{1}{27}$.

Ex. 11th $\frac{9}{10} : \frac{2}{3} = \frac{27}{20} = 1 \frac{7}{20}$.

Ex. 12th $\frac{9}{10} : \frac{1}{2} = \frac{9}{10} : \frac{3}{2} = \frac{18}{30} = \frac{3}{5}$.

Suppose a boat with eleven men
& a boy sails upon a river, & takes
a prize worth 136 L. The boy's
share is to be $\frac{1}{3}$ of a man's share
Q. how much each got.

$$136 : \frac{11}{3} = 136 : \frac{34}{3} = \frac{4081}{34} = 12$$

Answer. each man had 12 L & the
boy 4.

Of decimal Fractions Part of Notation

In decimal fractions an integer
is supposed to be divided into ten
equal parts and every one of
those ten parts are supposed to
be subdivided into other ten equal
parts &c.

The integer being thus
divided into 10, 100, 1000, 10000, &c.

equal parts these become the denominators to decimal fractions. As for instance $\frac{2}{10}$ $\frac{3}{100}$ $\frac{7}{1000}$ &c. But these denominators need not be set down; but only the numerators; & are distinguished or separated from whole numbers by a point or comma. Thus $5\frac{4}{10}$ is written 5.4 or 5,4 & $\frac{7}{10}$ is 0.7 & $35\frac{5}{100}$ is 35.05.

II Addition of Decimals.

Rule

Having placed the numbers to be added under one's another so that the commas or separating points may all stand in a direct line add the numbers as if they were integers & from the sum cut off so many decimals as are the most in any of the given numbers.

$$\begin{array}{r}
 \text{Ex. } 45.07 \\
 50.758 \\
 123.0057 \\
 74.702 \\
 24.8 \\
 \hline
 318.3357
 \end{array}$$

III. Subtraction of Decimals

Rule
To find the difference between

$$\begin{array}{r}
 45.30075 \text{ } \ominus \text{ } 74.284 \\
 \underline{45.375} \\
 28.909
 \end{array}$$

$$\begin{array}{r}
 437.5 \\
 89.657 \\
 \hline
 347.843
 \end{array}$$

$$\begin{array}{r}
 75.0034 \\
 57.875 \\
 \hline
 17.1284
 \end{array}$$

$$\begin{array}{r}
 562. \\
 93.5782 \\
 \hline
 468.4216
 \end{array}$$

III. Multiplication of Decimals

Rule
 Multiply them as if they were whole numbers then cut off or separate with a point or comma so many places of Decimal parts in the product as there are in both the factors together.

Ex.

3.024	32.12
2.23	24.3
69072	9536
6048	12848
6048	6424
674352	780.516

78.546	5745.
436	.0675
471276	28725
235638	46205
304184500	8470
92827656	3877775

If there be not so many figures in the product as there ought to be decimals by the rule, the defect must be supplied by prefixing cyphers to the product.

Ex.

$$\begin{array}{r}
 0.2365 \\
 0.2435 \\
 \hline
 11825 \\
 94695 \\
 4730 \\
 \hline
 0.05758775
 \end{array}$$

$$\begin{array}{r}
 0.0347 \\
 0.0236 \\
 \hline
 2082 \\
 1041 \\
 694 \\
 \hline
 0.00081892
 \end{array}$$

$$\begin{array}{r}
 57.056 \\
 .578 \\
 \hline
 456448 \\
 399392 \\
 285280 \\
 \hline
 32978368
 \end{array}$$

1781

V. Division of Decimals

Rule

Divide as if they were integers, then observe that the places of decimal parts of the divisor & quotient counted together must always be equal in number to those in the dividend.

Case I. When the places of parts in the divisor & dividend are equal, the quotient will be whole numbers.

$$\begin{array}{r} \text{Ex. I. } 0.45 \overline{) 295.75} \quad 35 \\ \underline{2535} \\ 4225 \\ \underline{4225} \\ 0 \end{array}$$

$$\begin{array}{r} \text{Ex. II. } 0.0078 \overline{) 4368} \quad 56 \\ \underline{390} \\ 468 \\ \underline{468} \\ 0 \end{array}$$

Case 2^d. When the places of parts
of the dividend exceed those in the
divisor cut off the excess for decimal
parts in the quotient.

Ex. I. $243 \overline{) 780.516} (32.12$

$$\begin{array}{r}
 729 \\
 \hline
 515 \\
 486 \\
 \hline
 291 \\
 243 \\
 \hline
 486 \\
 486 \\
 \hline
 000
 \end{array}$$

Ex. II. $436 \overline{) 34246.056} (78.546$

$$\begin{array}{r}
 3052 \\
 \hline
 3726 \\
 3480 \\
 \hline
 2380 \\
 2100 \\
 \hline
 2005 \\
 1744 \\
 \hline
 2616 \\
 2616 \\
 \hline
 0000
 \end{array}$$

(1782)

Case 3^o. When there are not so many places of parts in the dividend as in the divisor annex cyphers to the dividend to make them equal. Then will the quotient be whole numbers as in case 1.

Ex. I. $7.604 \overline{) 192.10.0} (25$

$$\begin{array}{r} 15360 \\ \hline 38420 \\ 38420 \\ \hline 00000 \end{array}$$

Ex. II $0.7875 \overline{) 441.0000} (560$

$$\begin{array}{r} 39375 \\ \hline 47250 \\ 47250 \\ \hline 00999 \\ \hline 47250 \end{array}$$

Case 4th. If after the division is finished there are not so many figures in the quotient as there ought to be places of parts by the general rule; supply their defect by prefixing cyphers to it.

Ex. I $957 \overline{) 7.25406} (0.00750$

$$\begin{array}{r}
 6699 \\
 \hline
 5550 \\
 4705 \\
 \hline
 7656 \\
 7656 \\
 \hline
 0000
 \end{array}$$

Ex. II $0.575 \overline{) 0.0007475} (0.0013$

$$\begin{array}{r}
 575 \\
 \hline
 1725 \\
 1725 \\
 \hline
 0000
 \end{array}$$

(1783)

To Reduce Vulgar fractions
into Decimals & the contrary

Rule

A vulgar fraction may be
reduced into decimals thus.

Annex cyphers to the nume-
rator & then divide it by the
denominator, the quotient will
be the decimal parts equi-
valent to the given fraction
or so near it as may be thought
necessary to approach.

Ex. 1 $\frac{3}{4}$ $\div 300$ (0.75

$$\begin{array}{r} 300 \\ 28 \\ \hline 20 \\ 20 \\ \hline 00 \end{array}$$

Ex. 11

$\frac{4}{7}$

7) 40000000 (0.571428571428

35

50

49

10

7

30

28

20

14

60

56

40

35

50

49

10

7

30

28

20

14

60

56

40

6c.

Ex. III

(1784)

~~11~~
13

13) 100000 (0.076923 076923

91
90
70
120
117

30
26
40
39

10
91
90
70
120
117

30
26

40
39

10
E.c.

Ex. IIII

7

7) 10000000 (0.142857 142857

7
30
20
20
14

60
56

40
35

50
49

10
7

30
20

20
14

60
56

40
35

50
49

10

Ec.

To Reduce a Decimal
to a Vulgar fraction of a
proposed denomination

Rule

Multiply the decimal by the
proposed denominator & from the
product cut off as many figures
as are contained in the proposed
decimal & you will have the
numerator corresponding to the
proposed denominator.

Ex. 1 0.75 proposed denominator 4

$$\begin{array}{r} 0.75 = \frac{3}{4} \\ 3 \overline{) 00} \end{array}$$

What is the value of 0.025 of one
pound sterling

$$\begin{array}{r} 0.025 = 15.6 \text{d} \\ 20 \\ \hline 16 \overline{) 500} \end{array}$$

To reduce $\text{£} 57142857142857$ Gr.

to 7^{ths}. ansr.

	7
3	9999999999999
	34

$\frac{4}{7}$

To find the value of 0.74722
of a pound Troy.

0.0719 out of 0.2 nearly.

	12
149444	74722
996664	20
1933200	27
13312	6656
79872	